QUIZ 4 - CALCULUS 3 (2021/4/15)
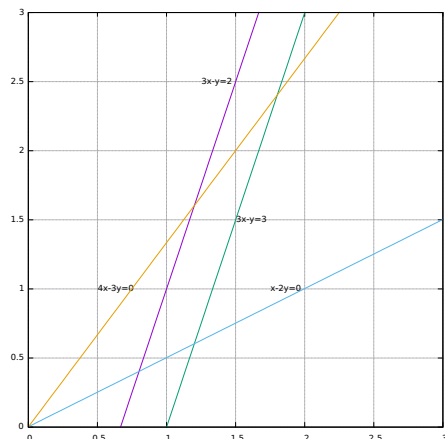
1. (14 points) Evaluate the double integral

$$\iint_R \exp(\frac{x+3y}{3x-y}) \, dA \ ,$$

where R is the region in the xy-plane bounded by 4 lines: $3x - y = 2$, $3x - y = 3$, $x - 2y = 0$, $4x - 3y = 0$.

Solution:

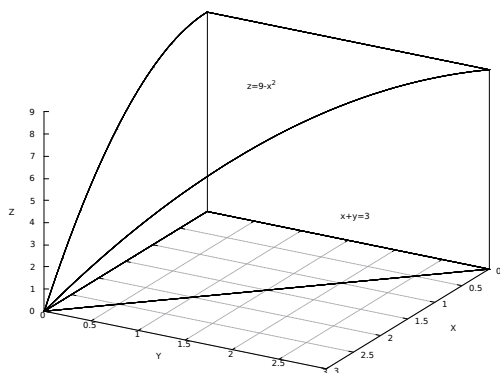Let $u = x + 3y, \ v = 3x - y$. (3 points)

Then we have $|J| = ||\partial(x,y)/\partial(u,v)|| = 1/10$. (3 points)

$$\iint_R \exp(\frac{x+3y}{3x-y}) \, dA \ = \ \int_2^3 \int_v^{3v} \exp(\frac{u}{v}) \, |J| \, dudv \ . \ \text{(4 points)}$$

$$\int_2^3 \int_v^{3v} \exp(\frac{u}{v}) \, |J| \, dudv = \frac{1}{10} \int_2^3 v \exp(\frac{u}{v})\Big|_{u=v}^{u=3v} dv = \frac{1}{4}(e^3 - e) \ . \ \text{(4 points)}$$

2. (6 points) Find a, b, c, d, e, and f in the following triple integral.

$$\int_0^3 \int_0^{9-x^2} \int_0^{3-x} f(x,y,z) \, dy \, dz \, dx = \int_a^b \int_c^d \int_e^f f(x,y,z) \, dz \, dx \, dy \ .$$

Solution: $a = 0, \ b = 3, \ c = 0, \ d = 3 - y, \ e = 0, \ f = 9 - x^2$. (1 point each)